Impact of Systematic Errors in Sunyaev-Zel'dovich Surveys of Galaxy Clusters

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(Dated: February 5, 2008)

Future high-resolution microwave background measurements hold the promise of detecting galaxy clusters throughout our Hubble volume through their Sunyaev-Zel'dovich (SZ) signature, down to a given limiting flux. The number density of galaxy clusters is highly sensitive to cluster mass through fluctuations in the matter power spectrum, as well as redshift through the comoving volume and the growth factor. This sensitivity in principle allows tight constraints on such quantities as the equation of state of dark energy and the neutrino mass. We evaluate the ability of future cluster surveys to measure these quantities when combined with Planck-like CMB data. Using a simple effective model for uncertainties in the cluster mass-SZ flux relation, we evaluate systematic shifts in cosmological constraints from cluster SZ surveys. We find that a systematic bias of 10% in cluster mass measurements can give rise to shifts in cosmological parameter estimates at levels larger than the 1σ statistical errors. Systematic errors are unlikely to be detected from the mass and redshift dependence of cluster number counts alone; increasing survey size has only a marginal effect. Implications for upcoming experiments are discussed.

Keywords: Sunyaev-Zel'dovich effect, dark energy theory, cosmological neutrinos

I. INTRODUCTION

Galaxy cluster surveys have the potential to place strong constraints on cosmological models, as has long been appreciated [1, 2, 3, 4, 5, 6, 7, 8]. The number of galaxy clusters as a function of cluster mass and redshift depends sensitively on both the gravitational growth factor and the comoving volume element as the universe evolves [9, 10, 11, 12, 13, 14]. In principle, large catalogs of thousands of galaxy clusters with masses and redshifts will constrain all cosmological parameters which affect either the growth factor or the rate of expansion at redshifts below $z \simeq 2$, where significant numbers of clusters have formed. In particular, this includes w, representing the equation of state of the dark energy, and $\sum m_\nu$, the total mass of the three neutrino species. Both of these quantities are of crucial importance for fundamental physics, and neither is well constrained by measurements of the cosmic microwave background power spectrum. Clusters will also give constraints on the total mass density of the universe, Ω_m, and the Hubble parameter h which are complementary to those from the microwave background.

In practice, the difficulty with using clusters as a cosmological probe lies in estimating their masses and in obtaining complete cluster samples. Past cluster surveys have relied on either X-ray or optical selection, but such surveys are complete only in a comparatively local region of the universe, and the connection between the observed optical richness or X-ray luminosity and the mass of the cluster is difficult to determine. With detection of clusters via their thermal Sunyaev-Zel'dovich (SZ) distortion of the microwave background now firmly established (see [15] for a fairly recent comprehensive list of detections), large cluster surveys with excellent completeness and improved selection functions are imminent [16, 17, 18, 19, 20]. (For reviews of the SZ effect, see [15, 21, 22, 23, 24, 25]). Since the SZ effect is essentially independent of redshift, SZ surveys will provide all galaxy clusters in the direction of a given sky region down to a limiting SZ distortion depending on cluster gas mass and pressure. Furthermore, the connection between SZ distortion and cluster mass is likely less sensitive to internal cluster physics than the corresponding connection for X-ray luminosity [26]. Experimental advances in detecting microwave fluctuations at small angular scales have raised hopes that we will soon have cluster catalogs in hand which will give meaningful information about fundamental physics.

While a number of papers so far have estimated the statistical errors in cosmological parameters from cluster surveys, relatively less emphasis has been placed on systematic errors and their impact (although see [8, 10, 27, 28]). An early important contribution [10] concluded that realistic uncertainties in cluster masses can lead to significant

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biases in cosmological parameters. Several subsequent papers, however, have implied that systematic errors may not be a limiting factor for cosmological conclusions [29, 30, 31]. It is clear that our ability to construct an unbiased estimator of cluster masses, whether through a correlation with their SZ signal, from gravitational lensing, or through other observables, will be the most important factor in determining the cosmological utility of cluster SZ catalogs. Conversely, as large SZ cluster surveys are now being planned, it is important to have a target accuracy for cluster mass determination: this affects not only observation strategies for the SZ signal, but also for the kinds of follow-up observations in other wave bands which will be required.

Our intention in this paper is to build on previous work [10] and focus on the effects of systematic errors on cosmological parameter extraction in the context of statistical constraints from upcoming microwave experiments. This paper aims to give a quantitative analysis of how well we need to understand cluster properties in order to realize the cosmological potential of these SZ cluster surveys. In particular, how small must systematic errors in cluster mass and redshift estimates be so that, for a cluster catalog of a given size, the bias in cosmological parameter determination due to the systematic errors is smaller than the statistical errors? We tackle this issue without relying upon any particular assumptions about the detailed cluster physics and survey-specific issues like cluster selection functions. Instead, we directly consider the uncertainties in the cosmologically relevant quantities, namely the cluster mass and redshift. This paramaterization effectively encompasses any uncertainty in cluster physics.

The following Section reviews the formalism for generating mock cluster catalogs from a given cosmological model, and displays the cluster distribution in both mass and redshift for several underlying cosmologies. Section III discusses some details of model fitting and error determination, using Monte Carlo and parameter-space search techniques. Then in Section IV we present results for the error in cosmological parameters due to biased cluster mass determination, for a number of different bias levels. We also display cluster mass and redshift distribution residuals between the best-fit model and the mock cluster catalog, for different assumptions about the cluster mass error. With large enough observed cluster catalogs, small differences in the observed distribution and the cluster distribution from the best-fit cosmological model can be statistically significant; we quantify the size of samples needed to detect these discrepancies. The concluding Section discusses these results in context of upcoming microwave cluster surveys.

II. DEPENDENCE OF CLUSTER EVOLUTION ON COSMOLOGICAL PARAMETERS

Ideally, a cluster Sunyaev-Zel'dovich survey will identify all the clusters in a certain angular region of the sky, $\delta\Omega$, and find their masses, M, and redshifts, z. The method of estimating the cluster distribution is well known (see, *e.g*, [27]). Consider the comoving mass function, which is the number density of clusters

$$\frac{dN}{dMdz}(M,z) = \delta\Omega \frac{dV}{dzd\Omega}(z)\frac{dn}{dM}(M,z) \tag{1}$$

within the comoving volume element $dV/dzd\Omega$ for a given solid angle $\delta\Omega$ on the sky. The mass function

$$\frac{dn}{dM}(M,z) \;=\; 0.315\frac{\rho_0}{M^2}\frac{d\ln\sigma_M}{d\ln M} \times \exp\left\{-\left|0.61 - \ln(\sigma_M D_z)\right|^{3.8}\right\} \tag{2}$$

describes the number density, n, of objects between masses of M and $M + dM$ at a given redshift z, where ρ_0 is the present density of matter. Eq. (2) is obtained from N-body cluster simulations [32] assuming a standard cosmological model. The dependence on mass comes through the spherical over-density

$${\sigma_M}^2 = \int_0^\infty dk(4\pi k^2)P(k)W^2(kR(M)), \tag{3}$$

where the matter power spectrum $P(k)$ is integrated within a sharply-defined spherical region of radius R, containing mass $M = 4\pi\rho_0 R(M)^3/3$ with a top-hat window function $W(x)$. The mass-independent quantities in Eqs. (1) and (2) are the volume factor $dV/dzd\Omega$ and the linear growth function $D_z = \delta(z)/\delta(0)$, where

$$\delta(z) = H(z)\int_0^{(1+z)^{-1}} \frac{da}{(aH(a))^3}.$$

For a given cosmology, the above equations completely determine the cluster abundance. Note that small changes in the mass fluctuations σ_M, specifically slight variations in numerics or in how the window function is defined, can lead to significant variations in dn/dM due to its exponential dependence on σ_M.

Neutrinos and dark energy have complementary effects, based on how they enter into Eq. (1). Dark energy has little effect on the primordial power spectrum, but directly affects the volume and growth factors. Neutrinos leave

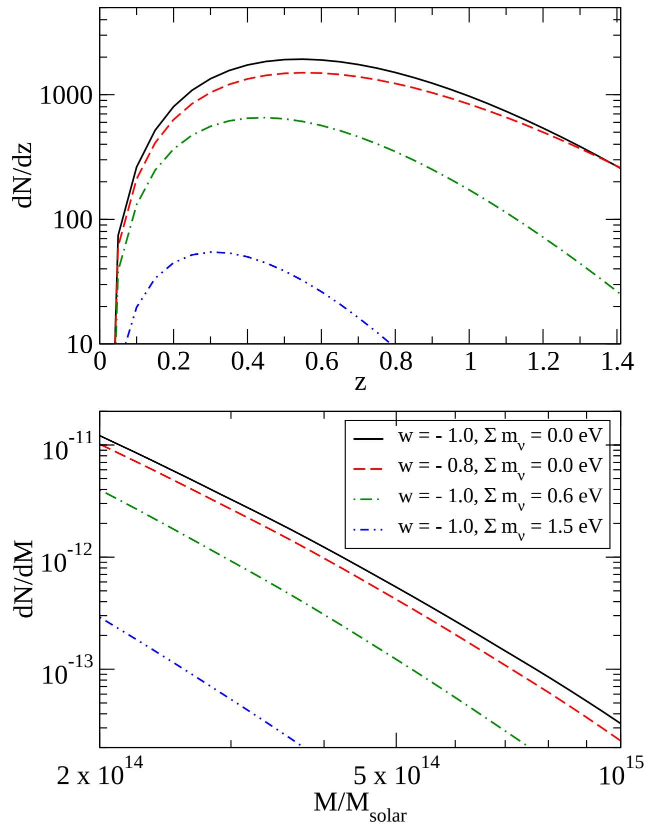

FIG. 1: The mass function integrated over mass (from $M_{lim} = 2 \times 10^{14} M_\odot$, top) and over redshift (bottom), for a survey area of $\delta\Omega = 200$ square degrees. Note that even a relatively small neutrino mass suppresses cluster formation strongly.

the volume factor unchanged, but suppress growth of fluctuations on scales smaller than the neutrino free-streaming length; total neutrino masses on the order of 0.5 eV can substantially suppress the power spectrum at scales relevant to cluster physics (*e.g.*, $k \gtrsim 0.02\ h/\mathrm{Mpc}^{-1}$).

Frequently, the cluster density is integrated over mass to yield the total cluster density in redshift only:

$$\frac{dN}{dz}(z; M_{lim}) = \int_{M_{lim}}^{\infty} dM \frac{dN}{dM dz}, \tag{4}$$

where the lower limit is an experimentally-determined limiting mass (which generally should depend on redshift [27]). This is the quantity plotted in the first plot of Fig. 1. However, a real survey will contain information about cluster masses through a flux-mass relation (see *e.g.* [33]), so neglecting the mass dependence loses information that can potentially be used to constrain cosmological parameters. Therefore, we also consider binning using the distribution function given in Eq. (1):

$$N_{ij} = \int_{M_i} dM \int_{z_j} dz \frac{dN}{dM dz}. \tag{5}$$

A given galaxy cluster survey will provide an estimate of N_{ij}; the remainder of this paper considers the impact of systematic mass errors in this estimate on the cosmological conclusions which can be drawn from it.

III. TREATMENT OF SYSTEMATIC ERROR

The previous section describes how, given a set of cosmological parameters, it is possible to obtain the theoretical distribution of clusters in mass and redshift. For real cluster catalogues, it is necessary to consider the converse procedure, taking a set of measured cluster counts as in Eq. (5) and constraining cosmological parameters from it.

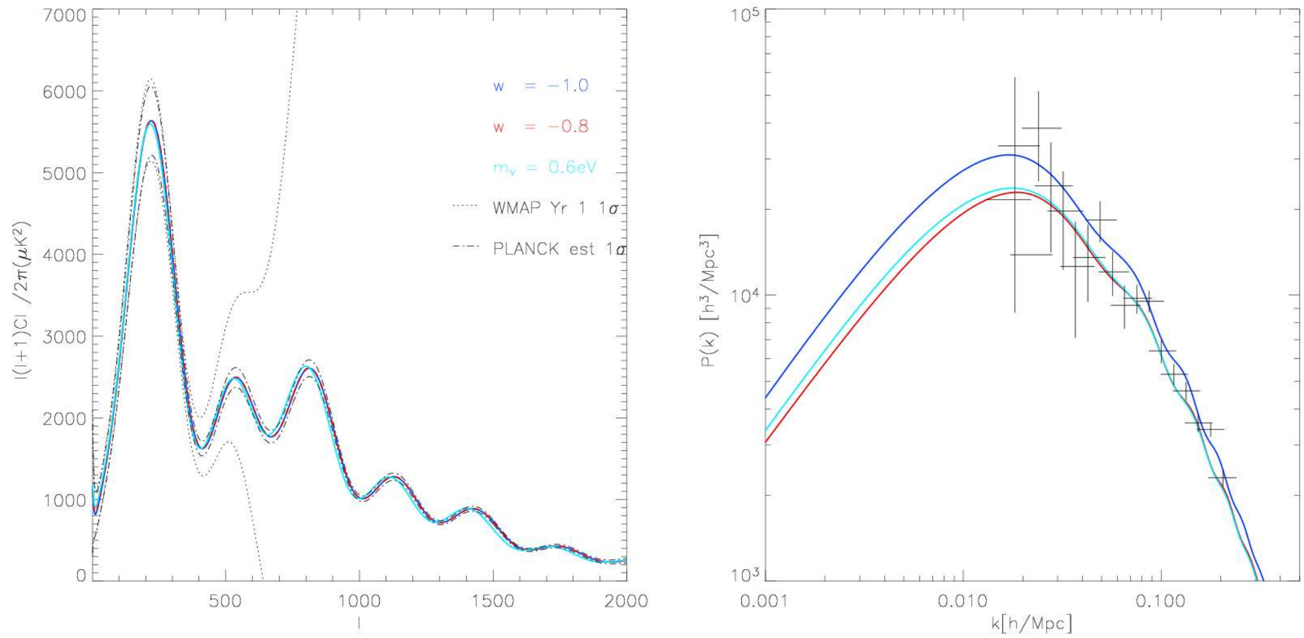

FIG. 2: Fiducial CMB and matter power spectra for models A,B and C as described in the text. Also shown are the error bars from the 1st year of WMAP data [48], projected errors for the Planck experiment, and data from the SDSS galaxy survey [49]

Markov Chain Monte Carlo techniques are well established in cosmology for constraining multi-dimensional parameter spaces [34, 35, 36, 37]. The cosmological information in an SZ survey depends on the minimum cluster mass probed by the survey and the survey's angular coverage. The following analysis considers a Planck-like measurement of the microwave background primordial power spectrum combined with an ACT-like Sunyaev-Zel'dovich cluster survey.

The utility of a given survey, however, depends on the extent to which systematic errors affect the inferred parameter values for a particular cluster catalogue. Cluster masses and redshifts are the fundamental quantities which can be determined for a given cosmological model from numerical simulations, and we will extract cluster mass and redshift estimates from upcoming observations. While redshifts can be measured to high accuracy with sufficient telescope time, determining masses poses a significantly harder challenge. Poorly understood galaxy cluster physics which modify a cluster's SZ signature can be viewed as a potential systematic error in mass estimates based on the SZ signal itself. Quantifying the effect of such systematic errors on constraining cosmological parameters is the main goal of this analysis.

Much of the cluster gas physics which is difficult to model—shock heating of intracluster gas, feedback from supernovae and active galactic nuclei [38], magnetic field turbulence [39, 40]—has the tendency to increase the SZ flux of a cluster relative to its mass, an effect observed especially in low-mass clusters (the "entropy floor" [41, 42, 43]). Therefore, the effect of these systematics is to make cluster masses inferred from their Sunyaev-Zel'dovich distortion larger than the actual cluster mass, boosting clusters from lower-mass bins into higher-mass bins, and increasing the total number of clusters in the sample above what would naïvely be expected from the sharp mass cutoff of Eq. (4). (Cooling flows, which we do not consider here, have the opposite effect, decreasing the SZ signature in relation to mass [44, 45].) A simple first-order model for the measured mass, motivated by numerical simulations [46], is

$$M = M_{real}(1+s), \tag{6}$$

where $s \geq 0$ is a constant. We neglect any statistical errors in the mass estimate, which in practice expand the error region in parameter space without moving its central value; such errors will also tend to increase the number of clusters in high-mass bins. (Lima and Hu [47] consider a scatter in the flux-mass relationship, which also produces significant effects.)

We calculate the CMB and matter power spectra using the CAMB [50] code for a cosmological scenario involving 7 parameters: $\Omega_b h^2, \Omega_{CDM} h^2, h, \tau, n_s, A_s$, and either w (assumed not to evolve with redshift) or $\sum m_\nu$. In addition, $\Omega_{total} = 1$ is held fixed. From a set of fiducial parameters, we calculate the temperature C_l and cluster N_{ij} assuming a Planck-style CMB experiment and an ACT-style SZ cluster survey over 200 square degrees out to $z = 1.4$ with

actual limiting mass $2 \times 10^{14} M_\odot$ and Poisson error bars. Systematic mass error is introduced by taking the ideal cluster binned data and relabeling the mass bins according to the prescription $M_{real} = M/(1+s)$, so that the total number of clusters in the survey is invariant. Then we determine which cosmological models are consistent with this altered cluster distribution by a standard Markov chain Monte Carlo calculation. Using the prescription given in [51] for CMB likelihoods and Poisson error bars for the cluster counts, we obtain the χ^2 between the best-fit model and the altered cluster distribution. Statistical errors scale with the square root of the number of clusters $\sqrt{N_{ij}}$, or equivalently the survey area $\sqrt{\delta\Omega}$.

We actually use the total number of clusters in each redshift bin, Eq. (4), as our observable for the cluster likelihood, rather than breaking the distribution into a number of mass bins as described by Eq. (5). This is because the dominant constraint on the parameters comes from the Planck-like CMB spectrum, and we find that breaking the cluster data into a number of mass bins does not significantly alter the error bars. The mass-binned distribution N_{ij} is still potentially useful for assessing goodness-of-fit for a given model, as discussed below. Real data would have a scatter in the number of clusters in each redshift bin consistent with Poisson errors; this scatter is neglected here so that the effect of any systematics on parameter determination is isolated from statistical error.

IV. NUMERICAL RESULTS

The fiducial cosmological parameters are chosen to give CMB spectra closely degenerate with the best fit spectrum from WMAP [48], consistent with projected error bars from Planck, and with the matter power spectrum from SDSS [49], as shown in Fig. 2. The effect of adding a bias to a fiducial ΛCDM scenario is shown in Fig. 3. A positive bias in the cluster mass estimate corresponds to believing that massive clusters are more numerous than is actually the case. The bias mimics an increased growth on cluster scales and roughly translates into an inferred increase in the density of clustering matter, increase in spectral tilt, or increase in overall amplitude. Including Planck-like CMB temperature data tightly constrains the tilt along with $\Omega_b h^2$ and $\Omega_c h^2$, with the latter implying that the bias might also drive an associated reduction in the inferred Hubble constant, H_0. The quantitative shift in the best fit cosmological parameters produced by systematic mass errors ($s = 0, 0.1, 0.2$) is shown in table I. We find a shift on the order of 1σ in all parameters except for σ_8, which moves roughly 6σ. However, the σ_8 values are all within the 1σ region obtained from the combined WMAP year 1 + SDSS matter power spectrum data ($\sigma_8 = 0.917^{+0.090}_{-0.072}$)[49]; this implies that the effect of the bias would not be significantly better constrained by including current matter power spectrum data. The best-fit linear $P(k)$ for the scenarios with and without bias are shown in Fig. 4.

Broadening the parameter space, the uncorrected bias would imply a larger number of clusters at all masses, as might be created by an upwards shift in the dark energy equation of state, or a reduction of neutrino density. In addition to the ΛCDM scenario, Table I shows parameter fits for systematic mass errors for three different models, two in which the neutrino mass is fixed at zero while w is allowed to vary, and one in which $\sum m_\nu$ is a parameter while $w = -1.0$ is fixed. Fig. 5 shows the error contours for the dark energy models A and B, and the massive neutrino scenario, model C, showing the shifts in the peak of the likelihood distribution as the amount of systematic error in the limiting mass increases is consistent with an attempt to lessen the suppression in the growth of structure produced by dark energy and massive neutrinos.

It is clear that the systematic misestimation of cluster masses can have a significant effect on the inferred cosmological parameters: a 10% shift in mass can yield parameter shifts on the order of 1σ (for the fiducial 200 square degree cluster survey in combination with a Planck-like CMB experiment). However, the shift in parameters produced by the bias is highly sensitive to the total number of clusters being fit. For example, a fiducial neutrino model with σ_8=0.78, gives only 476 clusters in the survey. In this case, a 10% mass bias has an insignificant effect on the best-fit parameters, and the bias only shows up in the excess χ^2 of the fit.

The goodness of fit of the best-fit model to the mass-biased cluster numbers is quantified by calculating the residuals

$$\chi_{ij} = \frac{N_{ij}^{\rm fit} - N_{ij}^{\rm data}}{\sqrt{N_{ij}^{\rm data}}} \tag{7}$$

where N_{ij} is the number of clusters in a given mass and redshift bin, so that $\chi^2 = \sum_{ij} (\chi_{ij})^2$. Fig. 6 shows the residuals plotted in mass and redshift bins for the two models.

One key consideration for the impact of systematics on the cosmological utility of a given SZ survey is whether the systematic shift in the measured bin counts is larger than the Poisson error for the number of clusters in that bin. If so, then the systematic distortion of the bin counts is detectable and can be measured; if the systematic shift per bin is smaller than the Poisson error, then it is not possible to diagnose the systematic error on a bin-by-bin basis. The bin residual χ_{ij} is normalized to the Poisson error in that bin, so if $\chi_{ij} = 1$ in a given bin, the systematic shift

Model	Error	Best-Fit Parameter						
		h	Ω_b	Ω_{CDM}	$\sum m_\nu$ (eV)	Ω_m	w	σ_8
Model ΛCDM	0%	$0.70\ ^{+0.01}_{-0.01}$	$0.050^{+0.001}_{-0.001}$	$0.25^{+0.01}_{-0.01}$		$0.30^{+0.01}_{-0.01}$		$0.93^{+0.01}_{-0.01}$
	10%	0.69	0.050	0.25	0.0	0.30	-1.00	0.97
∼1942 clusters	20%	0.69	0.051	0.26		0.31		0.99
Model A	0%	$0.70\ ^{+0.02}_{-0.03}$	$0.050^{+0.005}_{-0.002}$	$0.25^{+0.01}_{-0.03}$		$0.30^{+0.01}_{-0.03}$	$-1.00^{+0.05}_{-0.04}$	$0.93^{+0.03}_{-0.03}$
	10%	0.67	0.055	0.28	0.0	0.33	-0.91	0.92
∼1942 clusters	20%	0.65	0.060	0.30		0.35	-0.86	0.93
Model B	0%	$0.64\ ^{+0.03}_{-0.02}$	$0.060\ ^{+0.004}_{-0.004}$	$0.30^{+0.02}_{-0.02}$		$0.36^{+0.02}_{-0.03}$	$-0.80^{+0.06}_{-0.09}$	$0.88^{+0.03}_{-0.02}$
	10%	0.63	0.062	0.32	0.0	0.38	-0.77	0.89
∼ 2248 clusters	20%	0.62	0.062	0.32		0.38	-0.77	0.92
Model C	0%	$0.65\ ^{+0.01}_{-0.01}$	$0.056\ ^{+0.001}_{-0.002}$	$0.32\ ^{+0.01}_{-0.01}$	$0.60\ ^{+0.07}_{-0.13}$	$0.38\ ^{+0.02}_{-0.01}$		$0.84\ ^{+0.01}_{-0.02}$
	10%	0.65	0.056	0.33	0.50	0.39	-1.00	0.90
∼ 1601 clusters	20%	0.64	0.056	0.33	0.50	0.40		0.92

TABLE I: Four models showing the change in the best fit cosmological parameters for 0%, 10%, and 20% systematic errors in the mass. In all cases, the survey area is $\delta\Omega = 200$ square degrees with a limiting mass of $M_{lim} = 2 \times 10^{14} M_\odot$ and a single mass bin at each redshift slice. Models A and B fix the neutrino mass at zero, while model C allows the neutrino mass to vary while fixing w. The 1σ uncertainty in the fiducial parameters for a Planck-like CMB temperature spectrum plus cluster constraints are given to compare the against the shifts in parameters from the systematic error. The σ_8 values for all models lie within the 2σ error region obtained from WMAP [48].

in cluster counts is the same size as the 1σ statistical error. The statistical errors scale trivially with the square root of the survey area $(\delta\Omega)^{1/2}$. In the models we have studied,

$$\frac{|\chi_{ij}|}{\sqrt{\delta\Omega}} \lesssim \begin{cases} 0.017 & \text{Model A} \\ 0.015 & \text{Model B} \\ 0.032 & \text{Model C} \end{cases} \tag{8}$$

To attain $\chi_{ij} = 1$ in the bins with the largest count distortions requires a survey on the order of 3500 square degrees for model A, 4300 square degrees for model B, and 980 square degrees for model C. While Planck will cover the entire sky with perhaps 35000 square degrees usable for cosmology, it has a higher cluster mass detection threshold ($\simeq 5 \times 10^{14}\, M_\odot$) due to its relatively large beam [52]; each Planck cluster bin will have fewer clusters, and the values in Eq. (8) are significantly smaller.

Figure 6 displays the normalized residuals χ_{ij} per bin. Not surprisingly, the largest values are at the lower end of the cluster mass range, where the bins have the largest populations. This simply reflects the fact that the largest mass clusters represent the peaks in the initial mass distribution, and their population is more sensitive to small changes in cosmology. Note that in all cases considered, the systematic discrepancies are not randomly distributed throughout the bins, but rather have a coherent structure. The condition $\chi_{ij} = 1$ should be viewed as a rough estimate of the overall size of the distortions. By modeling particular coherent patterns of discrepancy over the mass and redshift bins, it may be possible to diagnose particular systematic distortions even if every individual bin has a systematic shift smaller than a 1σ statistical error for that bin.

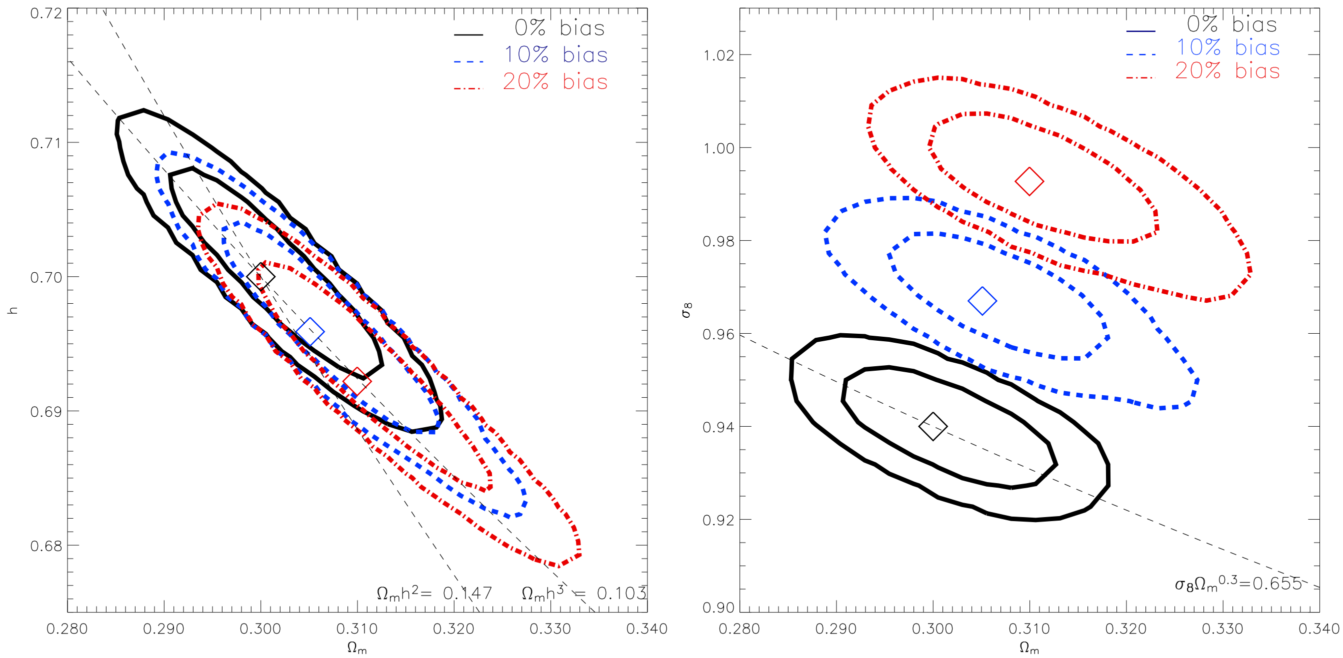

FIG. 3: The effect on key cosmological parameters of 0%, 10% and 20% positive bias in the cluster mass estimate for the ΛCDM model.

V. DISCUSSION

The compilation of large galaxy cluster catalogs selected via Sunyaev-Zel'dovich distortion of the microwave background will be a reality within the next few years, and these data sets will open a new realm of cosmological inquiry. The importance and potential impact of these measurements is widely recognized. A recent report by the Interagency Working Group on the Physics of the Universe, based on the National Research Council's 2002 Turner Commission Report, stated that a highest priority investigation should be a coordinated effort to use the number of galaxy clusters observed in SZ surveys and X-ray observations as a dark energy probe [53].

The goal of this paper is to provide a realistic assessment of how well galaxy cluster physics will need to be understood for the upcoming cluster surveys to realize their cosmological potential. The cluster properties which are most easily connected to predictions from cosmological simulations are redshift, mass, and peculiar velocity. Redshift is, with sufficient optical telescope resources, measurable to high accuracy. We have therefore not considered systematic redshift errors here, although for surveys which rely on photometric redshifts rather than spectroscopic ones systematic redshift errors may not be negligible. The cluster mass distribution is most often considered as the basic relation which future SZ surveys will measure, and here we have focused on systematic mass errors. The cluster mass is not directly measurable via the SZ distortion on an individual cluster basis, and the relation between SZ flux and mass must be assumed, extracted from simulations, or measured in some other way. Two general classes of techniques are currently under study: "self-calibration" of the cluster mass-flux relation directly from the SZ survey [47, 54], and use of other cluster observables like X-ray emission or weak lensing shear.

It is quite reasonable to expect that a combination of data sets and analysis techniques will lead to reasonable cluster mass estimators. How good will these estimators need to be? Here we have presented a model calculation showing that systematic biases in cluster mass estimates at the 10% level are enough to shift cosmological parameter estimates by more than the statistical 1σ error bars for some parameters, particularly the dark energy equation of state w. This assumes that cosmological parameters will be constrained using a Planck-like measurement of the primary microwave background fluctuations. One might hope that the distribution of cluster masses and redshifts would be sufficiently altered by systematic misestimates of cluster masses that the systematic error would be detectable in the cluster distribution itself; we show here that this is likely to be only marginally possible with upcoming cluster SZ surveys.

Our conclusions from this study are cautiously optimistic. It is clear that upcoming SZ cluster surveys will be in the regime where systematic errors due to cluster astrophysics will be important for interpreting the results. On

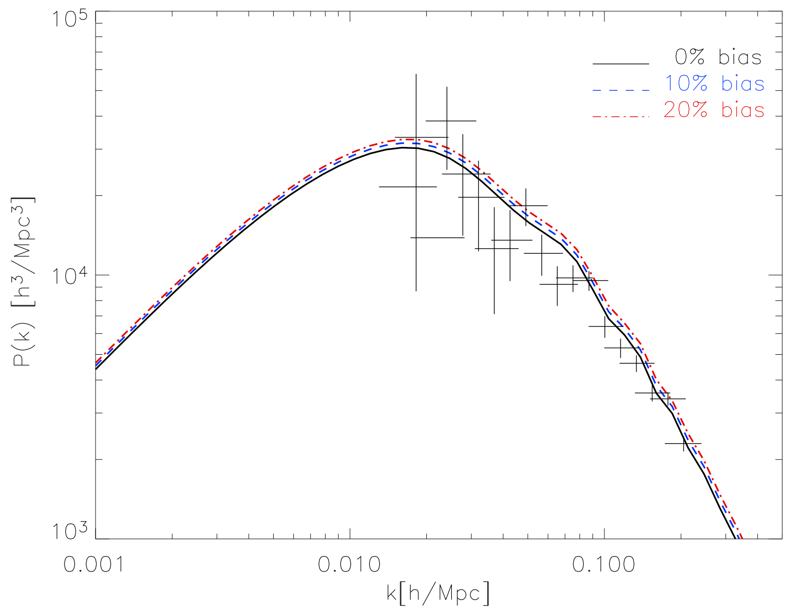

FIG. 4: The effect on the best fit matter power spectrum P(k) of imposing 0%,10% and 20% cluster mass misestimation in the ΛCDM model. As one would expect the effect of the bias is analogous to a boost in power on cluster scales.

the other hand, with a variety of potential observations and techniques for diagnosing and accounting for systematic errors, we can plausibly expect to reduce the impact of systematic errors on cosmological conclusions to the level of statistical ones. This will by no means be simple; cluster mass estimates for a large sample with no more than 10% bias is hard to do. (Note that the bias in the cluster mass measurements, and not the size of the scatter in the measurements, is the relevant error to consider.) SZ cluster catalogs must be conceptualized as the basis for a range of other complementary measurements and calculations which, taken as a whole, can contribute significant cosmological constraints.

The challenges of controlling systematic errors in cluster mass estimates also prompts consideration of alternate possibilities for extracting cosmology from SZ catalogs. The underlying difficulty with cluster masses is that they are only indirectly probed by SZ measurements, but the directly measured SZ flux is not easily related to cosmological properties (and the clusters will likely not even be precisely flux-selected [31]). Furthermore, most astrophysical processes in clusters shift the SZ signal towards larger fluxes, so mass estimates from the SZ signal are generally biased high, and a careful accounting for all contributing effects must be undertaken. The number of clusters as a function of redshift or mass is very sensitive to this unavoidable difficulty. Other cluster observables are potentially less sensitive to mass biases in an SZ cluster catalog. The distribution of clusters on the sky and in redshift is one obvious possibility [7, 29, 54, 55]. While selection biases in large-scale structure surveys have been studied extensively in the context of galaxy catalogs, relatively little analysis has been done on selection biases in corresponding SZ-selected cluster catalogs.

Another alternative may be to use the kinematic SZ effect to construct cluster peculiar velocity catalogs [56]. The kinematic SZ effect is smaller amplitude than the thermal SZ signal, and its frequency dependence is nearly the same as the blackbody primary microwave background fluctuations, so its detection requires higher sensitivity measurements and sophisticated techniques for separating the kinematic SZ signal from other signals and noise sources. But its advantage is that few systematic errors are correlated strongly with the kinematic SZ signal, and unbiased estimates of cluster peculiar velocities are possible in principle [57, 58]. This is a promising alternate route to cosmological constraints from SZ cluster catalogs which is less susceptible to systematic errors intrinsic to cluster properties [59, 60].

The goal of this paper is to shift the focus of the discussion about SZ surveys from their abundant potential to provide interesting constraints on cosmology, which has been well demonstrated, to the level at which systematic errors must be controlled. Some systematic errors are unavoidable, due to intrinsic astrophysical uncertainties in galaxy clusters, while others will result from practical limitations on given experiments and on algorithms for separating different signal components given a limited number of frequency channels and limited angular resolution. Here we advocate, in addition to detailed study of these individual effects, examining the impact of all of these using an effective

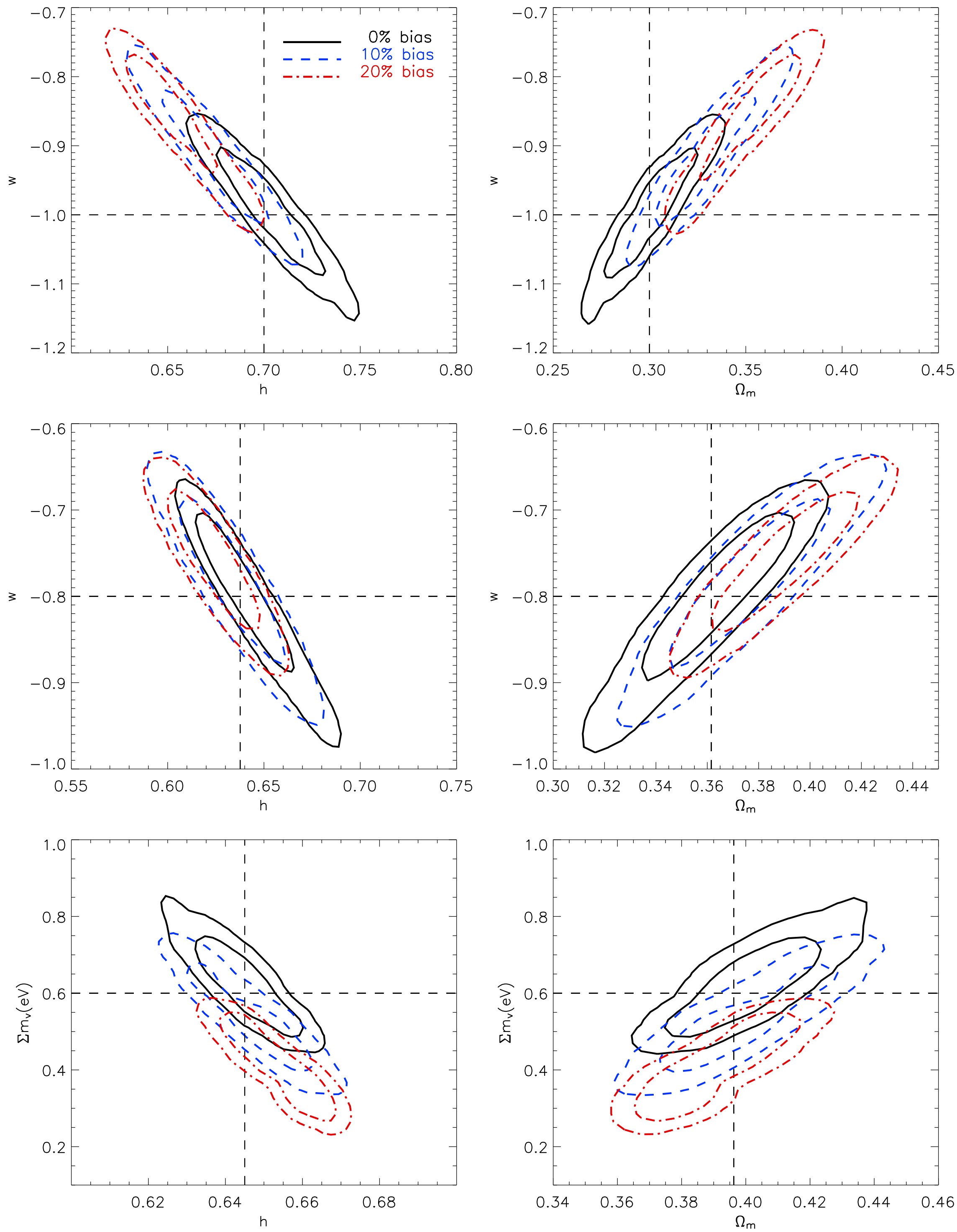

FIG. 5: Error contours for dark energy models, A (top row) and B (middle), and massive neutrino model, C (bottom) for 0%, 10% and 20% systematic error in cluster mass.

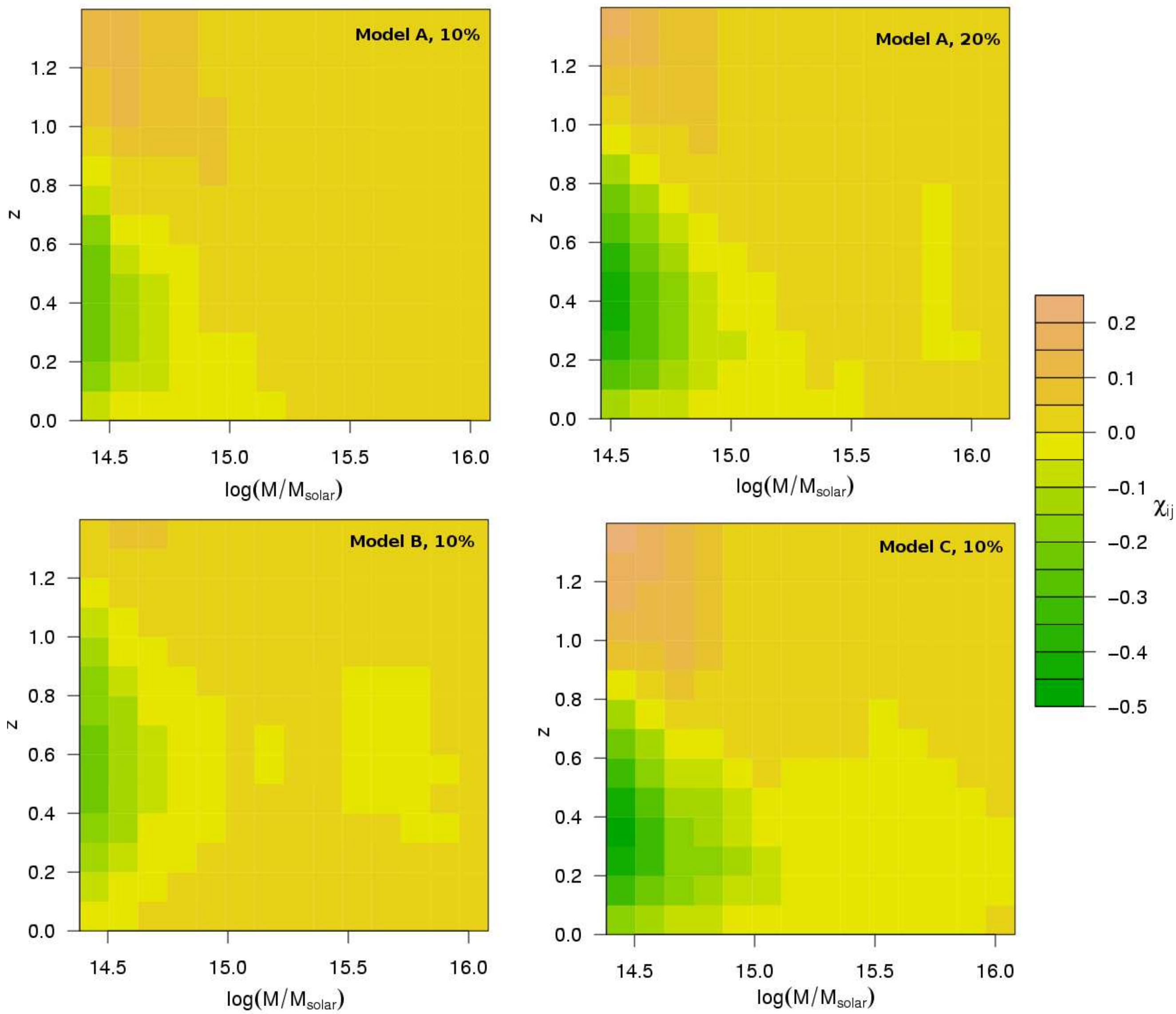

FIG. 6: The distribution of redshift and mass bin-wise residuals χ_{ij} Models A (top row, 10% and 20% mass shift), B and C (bottom row, 10% mass shift). Note that the distribution of positive and negative residuals indicates the change in the shape of the mass function due to the change in cosmological parameters. If the bin-wise residuals are sufficiently large, this pattern may be a means to detect the presence of systematic error in mass estimates.

model of systematic errors in the ultimate physical quantities used in constructing cosmological tests, namely cluster masses and redshifts. We have considered a simple proportional shift in inferred cluster mass relative to the actual cluster mass; clearly more complex models may be useful. We have also looked only at a few cosmological models, due to the computational difficulty of surveying wide sets of models each with its own set of Markov Chain Monte Carlo calculations. One highly useful direction for future work is constructing much faster approximations to the matter power spectrum (especially in the massive neutrino case), which could greatly increase the Markov Chain efficiency. Such approximations have already been proven for the microwave background primary power spectrum [35, 61, 62], which is more complicated than the matter power spectrum. With such tools in hand, it will be possible to perform far more exhaustive calculations of systematic effects than those presented here, including a wider range of underlying cosmological models, different models of systematic effects, and combinations of other sources of cosmological data. This kind of extensive analysis is, in our opinion, essential for planning observational programs complementary to the SZ surveys currently undertaken, programs which will be demanding in time and resources yet which hold the key to maximizing the return on our investment in SZ observations.

Acknowledgments

MRF thanks Jack Hughes and Licia Verde for their many helpful comments on the chapter of his thesis that parallels this paper, and Antony Lewis for help in using code from CAMB and CosmoMC. The authors thank Raul Jimenez, Lyman Page and an anonymous referee for useful comments. This work has been partly supported by the NSF award 0408698 for the ACT project. The work of RB was supported by funds from Cornell University. This research used computational facilities supported by NSF grant AST-0216105.

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